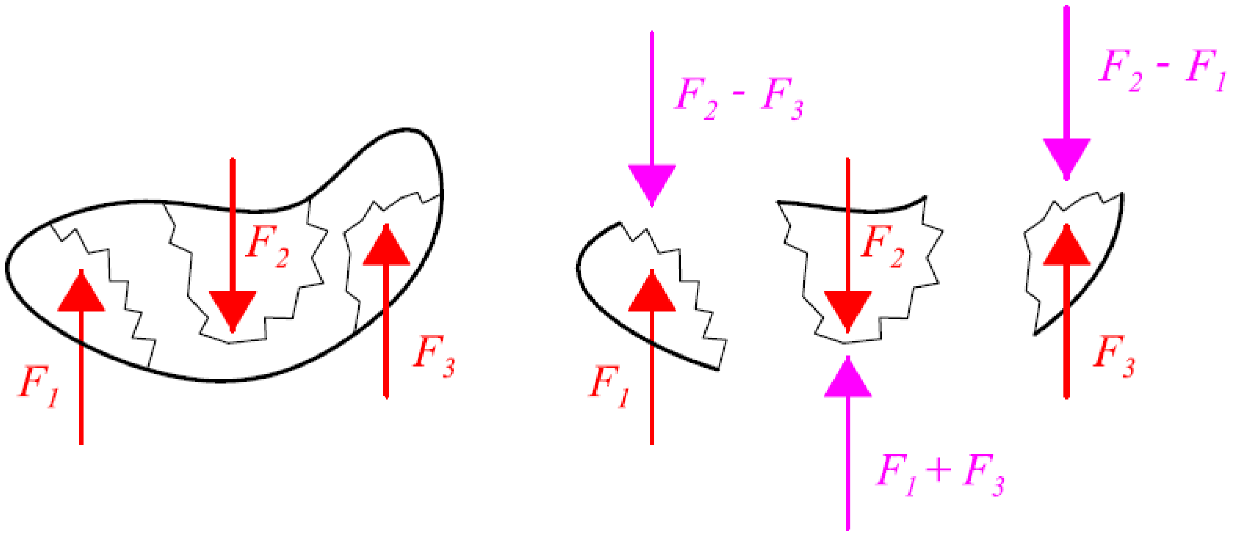
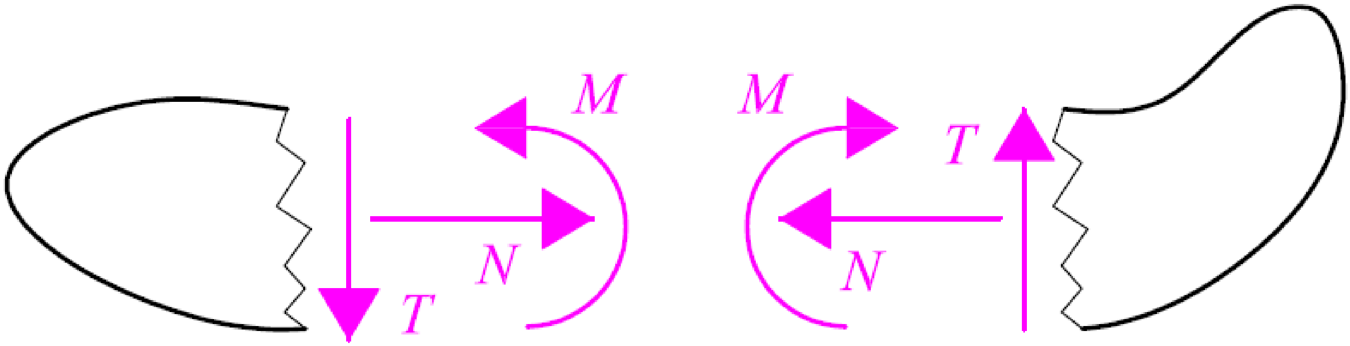
Statique des structures

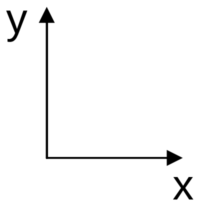


Forces en equilibre appliqué à un corps

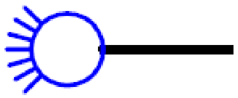


Forces internes à un solide

Symboles des liaisons et degrés de liberté éliminés

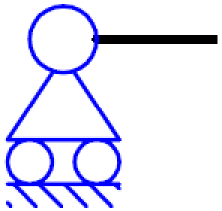


pivot



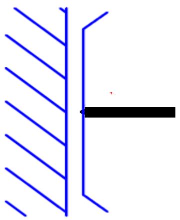
$\dot{x} = 0$
$\dot{y} = 0$

pivot - glisseur



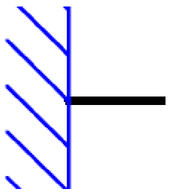
$\dot{y} = 0$

glisseur



$\dot{x} = 0$
$\dot{\theta} = 0$

encastre



$\dot{x} = 0$
$\dot{y} = 0$
$\dot{\theta} = 0$

<u>structure iso, iper, ipo-statique?</u>

2n – p – v

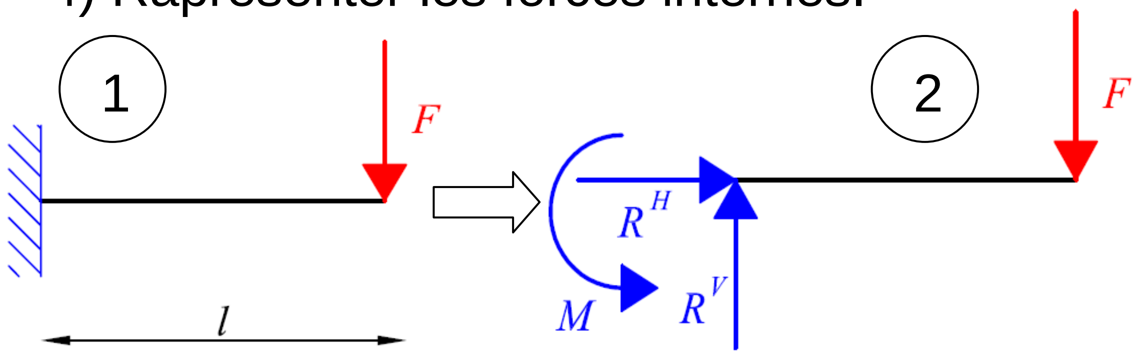
= 0 : iso, solution possible avec equations d’equilibre

< 0 : iper, solution possible avec elimination d’un liaison

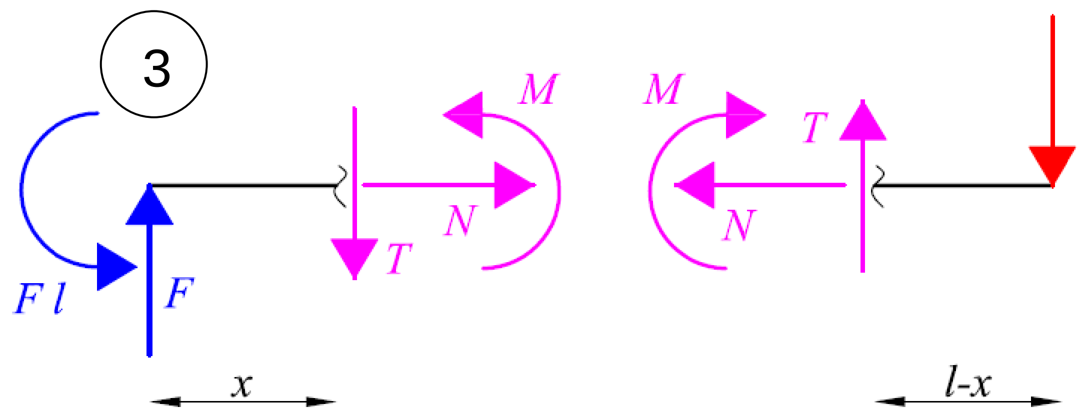
> 0 : ipo, aucun solution possible, la structure n’est pas statique

Calcul d'une structure :

1) Verifier si la structure est ipo, iper, isostatique. Choisir la methode de resolution.
2) On trouve le reactions des liaisons.
3) Couper chaque poutre pour trouver les forces internes.
4) Rapresenter les forces internes.



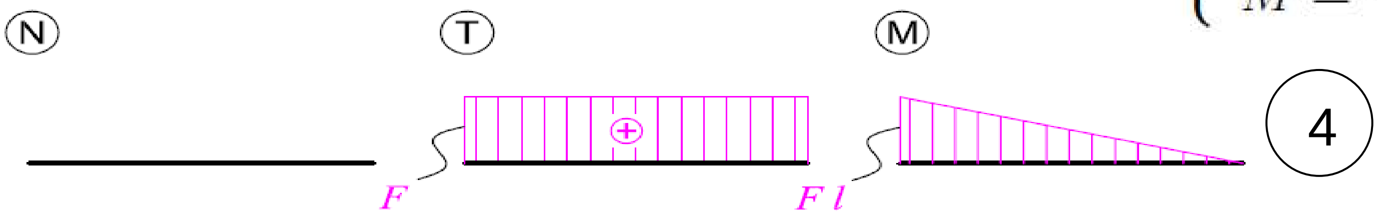
$$\begin{cases} R^H = 0 \\ R^V - F = 0 \\ M - F \cdot l = 0 \end{cases} \Rightarrow \begin{cases} R^H = 0 \\ R^V = F \\ M = F \cdot l \end{cases}$$



$$\begin{cases} N = 0 \\ -T + F = 0 \\ F \cdot l + M - F \cdot x = 0 \end{cases}$$

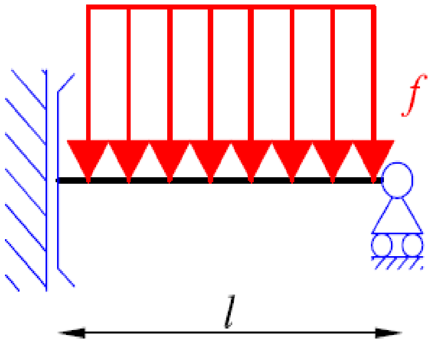
$$\Downarrow$$

$$\begin{cases} N = 0 \\ T = F \\ M = -F \cdot l + F \cdot x = 0 \end{cases}$$

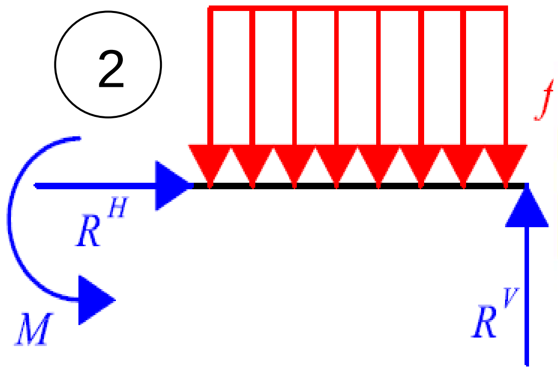


1

Autre exemple avec charge distribué :

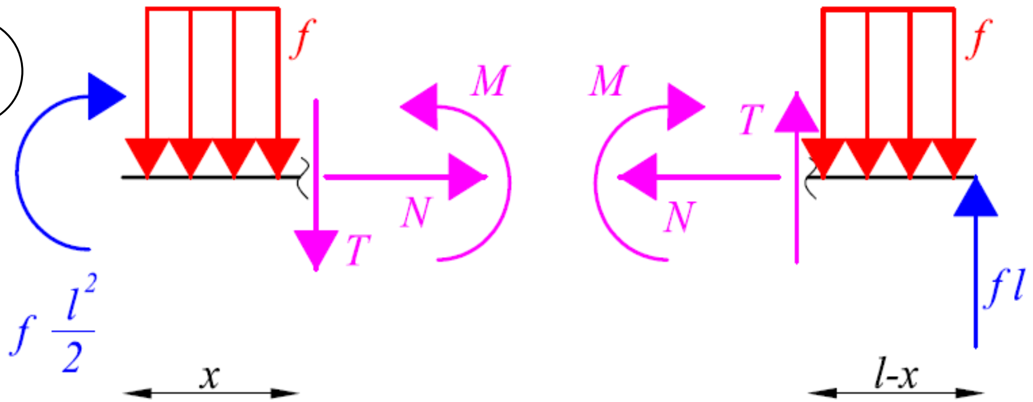


2



$$\left\{\begin{array}{l} R^H = 0 \\ R^V - f \cdot l = 0 \\ M + f \cdot l \frac{l}{2} = 0 \end{array}\right. \Rightarrow \left\{\begin{array}{l} R^H = 0 \\ R^V = f \cdot l \\ M = -f \frac{l^2}{2} \end{array}\right.$$

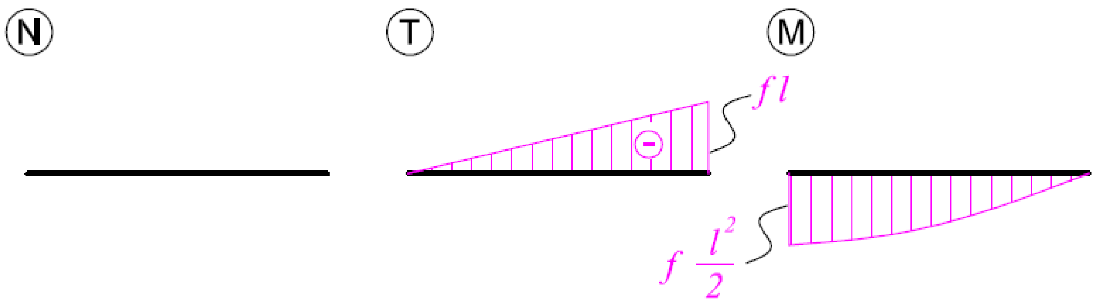
3



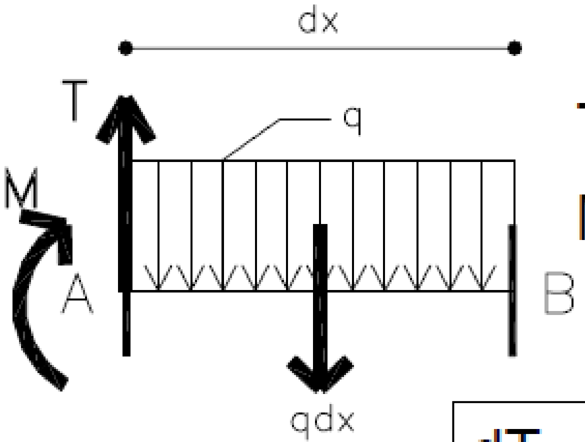
$$\Rightarrow \left\{\begin{array}{l} N = 0 \\ -T - f \cdot x = 0 \\ M - f \frac{l^2}{2} + f \cdot x \frac{x}{2} = 0 \end{array}\right.$$

4

$$\left\{\begin{array}{l} N = 0 \\ T = -f \cdot x \\ M = f \frac{l^2}{2} - f \frac{x^2}{2} \end{array}\right. \Rightarrow$$



L'equation de la ligne élastique :



$$T_1 = T + dT \quad \Rightarrow \quad T_1 = T + q\,dx$$

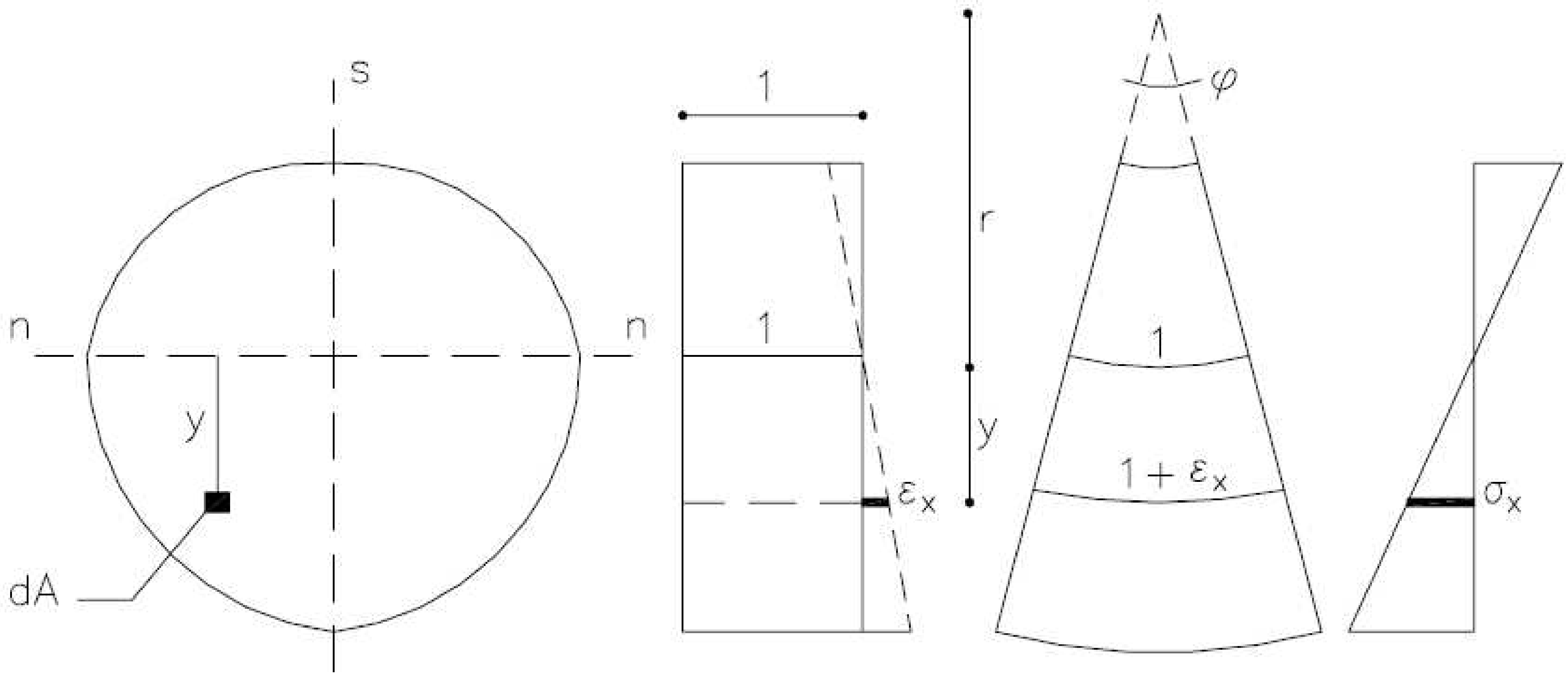
$$M_1 = M + dM \quad \Rightarrow \quad M_1 = M + T\,dx - q\,dx\,dx/2$$

$$\boxed{dT = -\,q\,dx \qquad dM = T\,dx}$$

hypotheses :

1) Aprés la deformation les sections sont encore planes
2) Petites deformations
3) Loi de Hooke valable $\sigma_x = E\,\varepsilon_x$

On considere une poutre en flexion :



$$\sigma \int_A y^2 \, dA = M \quad \Rightarrow \quad \sigma = \frac{M}{J}$$

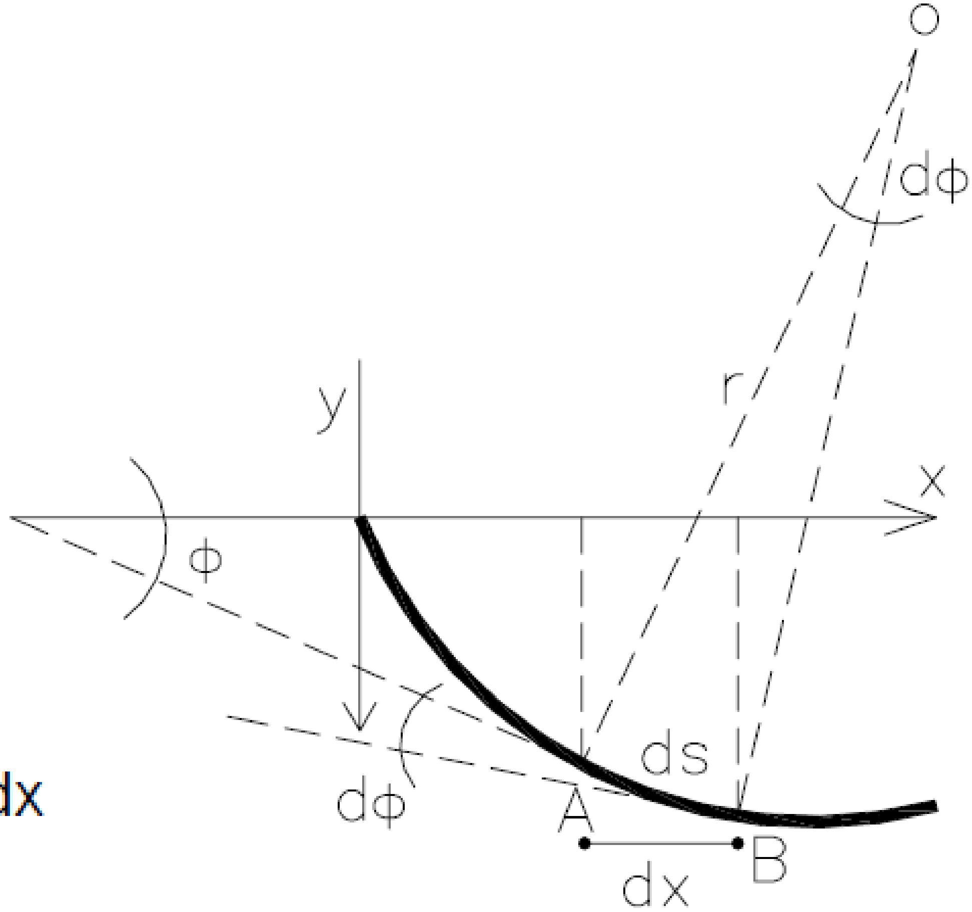
Pour petits deformations la courbure est $\dfrac{1}{r} = \dfrac{M}{EJ} = \varphi$

Si la deflexion est petite:

$ds = r\, d\Phi$

$ds \approx dx; \quad \Phi \approx tg\, \Phi = dy/dx$

$$\frac{1}{r} = -\frac{d^2y}{dx^2} \Rightarrow \boxed{EJ\frac{d^2y}{dx^2} = -M}$$



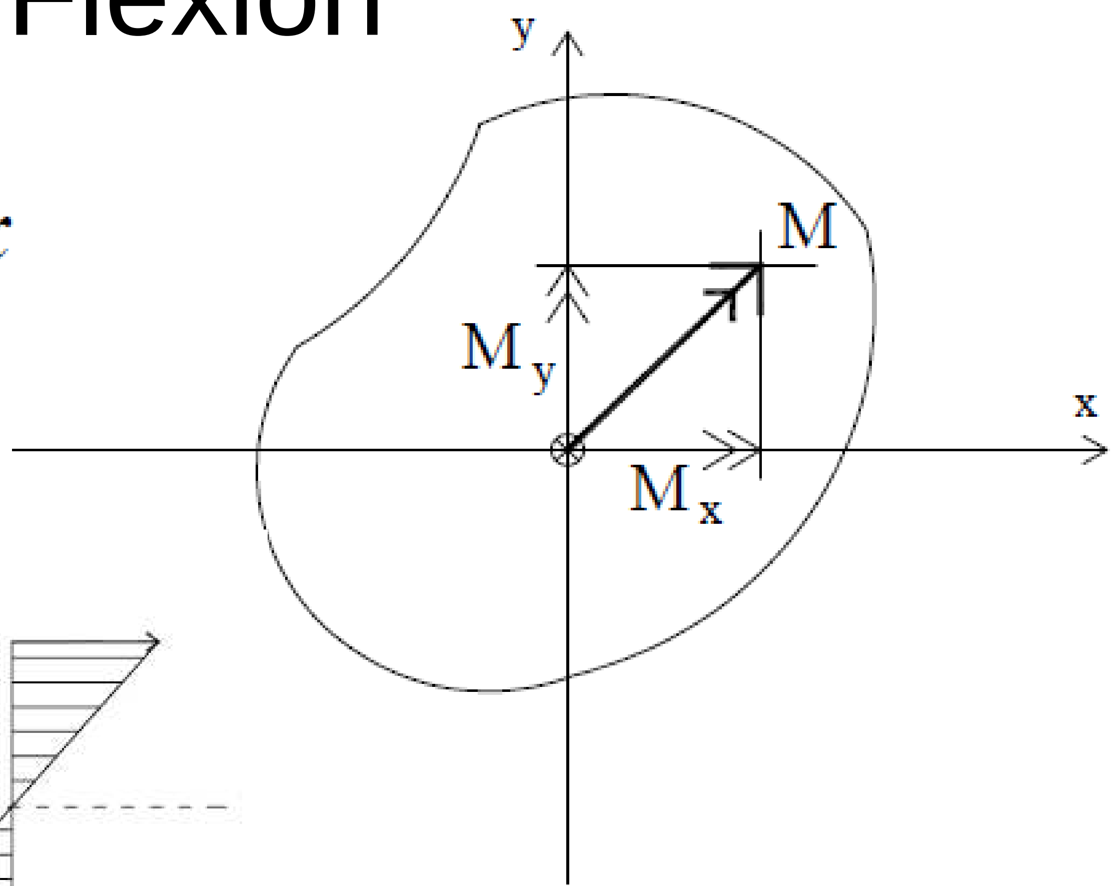
Si

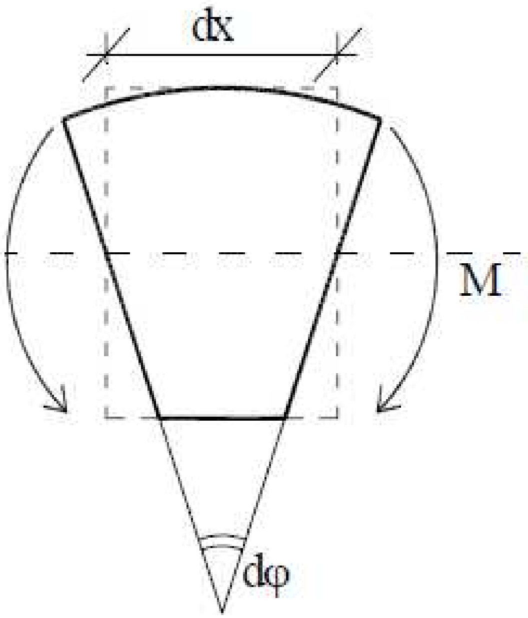
$dT = -q\, dx \qquad dM = T\, dx$

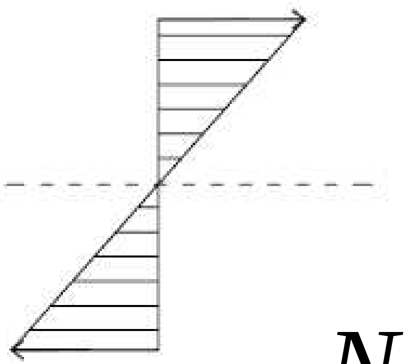
$$\boxed{\begin{aligned} EJ\frac{d^3y}{dx^3} &= -T \\ EJ\frac{d^4y}{dx^4} &= -q \end{aligned}}$$

Flexion

$$\sigma = \frac{M_x}{I_x} y - \frac{M_y}{I_y} x$$



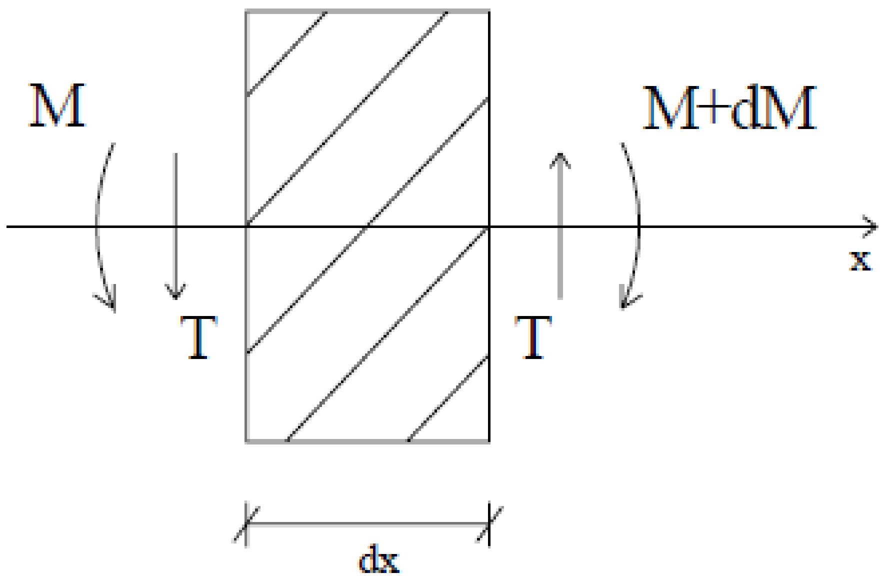


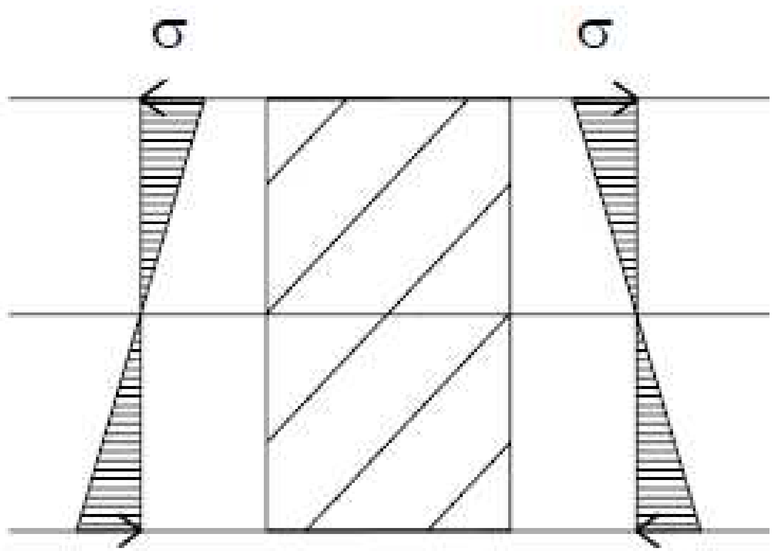


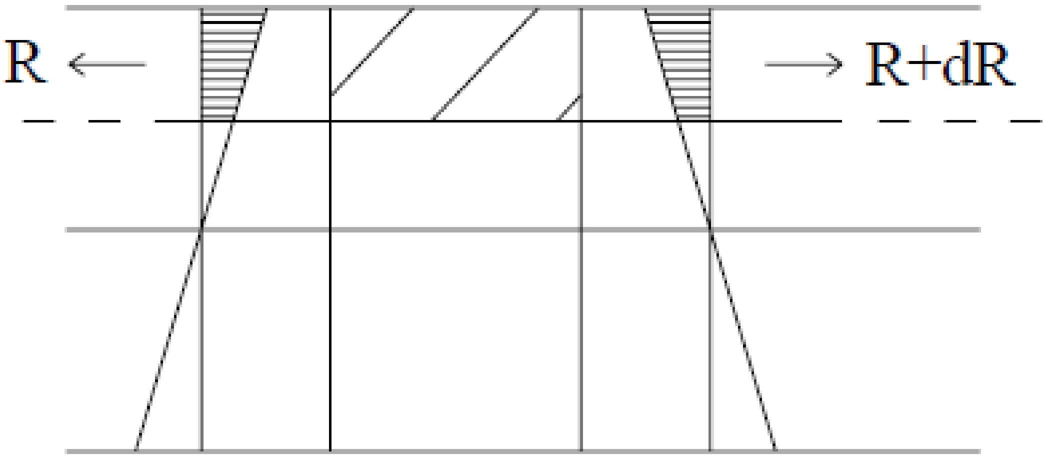
$$\varepsilon_{33} = \frac{N}{EA} + \frac{M_1}{EJ_{11}} x_2 - \frac{M_2}{EJ_{22}} x_1$$

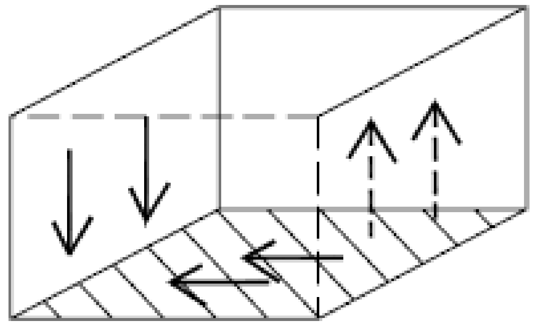
Cisaillement
(formule de Jouraski)

$$dM = T \cdot dx$$





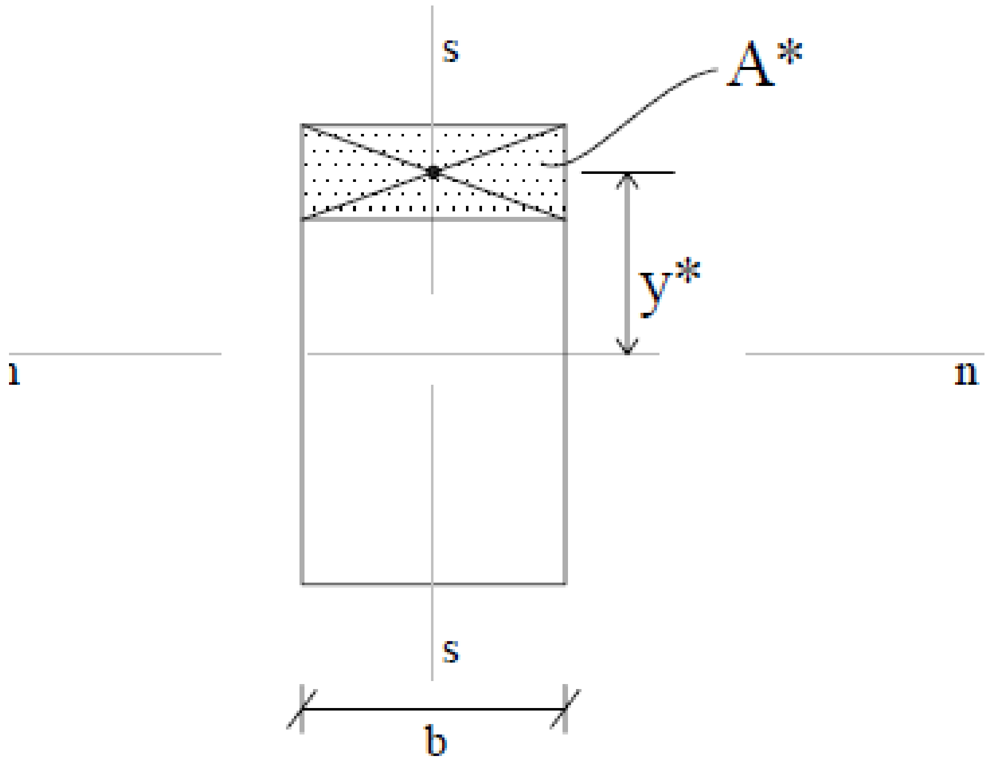




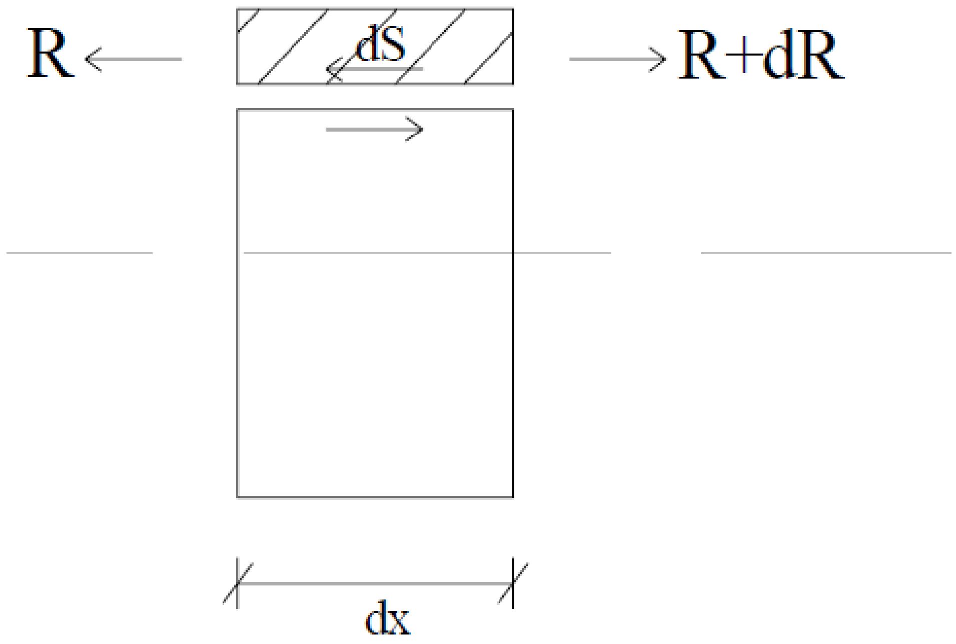
$$dR = dS$$

$$dS = \tau(b \cdot dx)$$

$$R = \int_{A^*} \sigma \cdot dA = \int_{A^*} \frac{M \cdot y}{I_n} dA = \frac{M}{I_n} \int_{A^*} y \cdot dA = \frac{M}{I_n} S^*$$



$$R + dR = \left(M + dM\right)\frac{S^*}{I} \rightarrow dR = dM\frac{S^*}{I} = T \cdot dx\frac{S^*}{I}$$

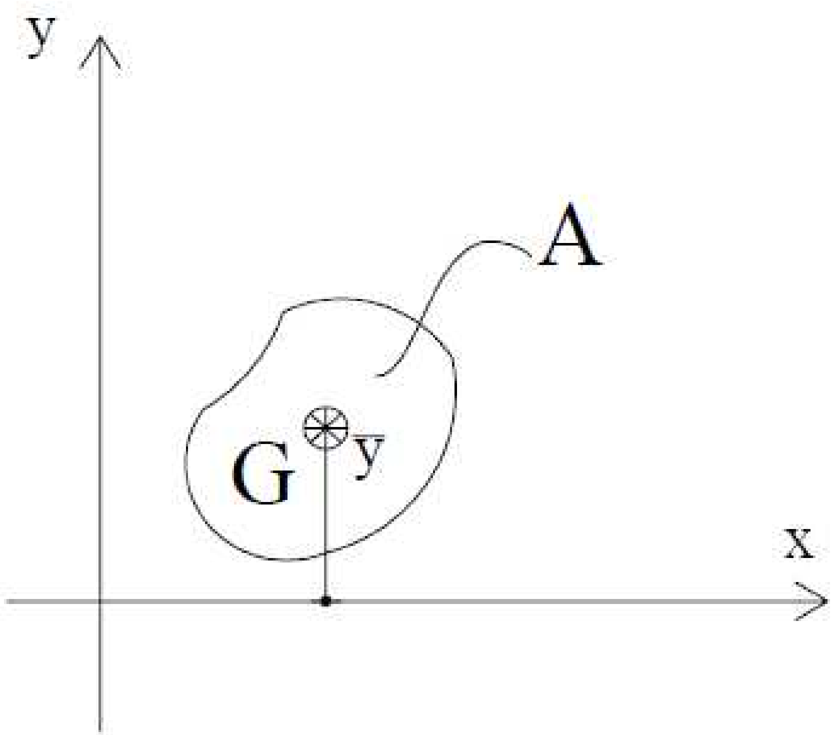


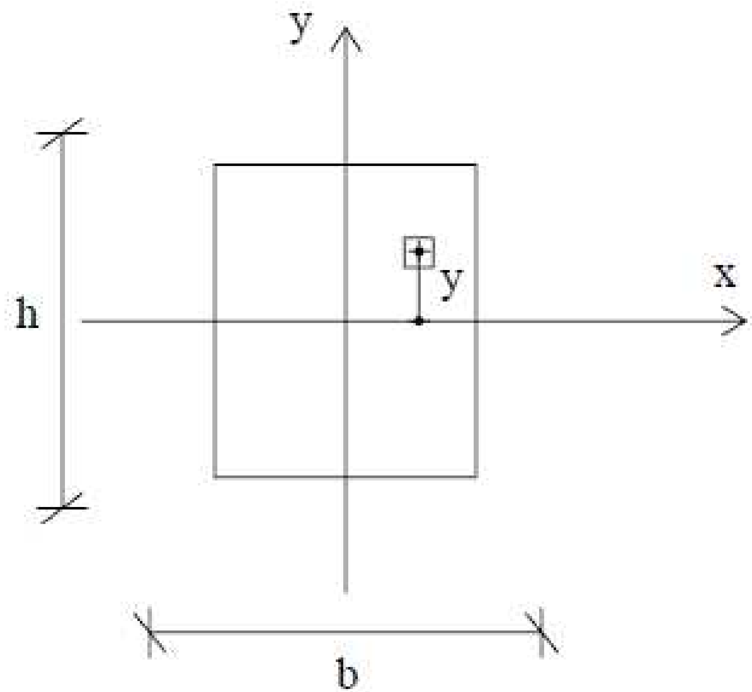
$$dR = dS \rightarrow Tdx\frac{S^*}{I_n} = \tau \cdot b \cdot dx$$

$$T\frac{S^*}{I_n} = \tau \cdot b \rightarrow \tau = T\frac{S*}{I_n}$$

Le moment statique et d'inertie

$$A = \int_A dA \qquad\qquad S_x = \int_A y \cdot dA = \bar{y} \cdot A$$





$$I_x = \int_A y^2 \, dA$$

rectangle $\Rightarrow$

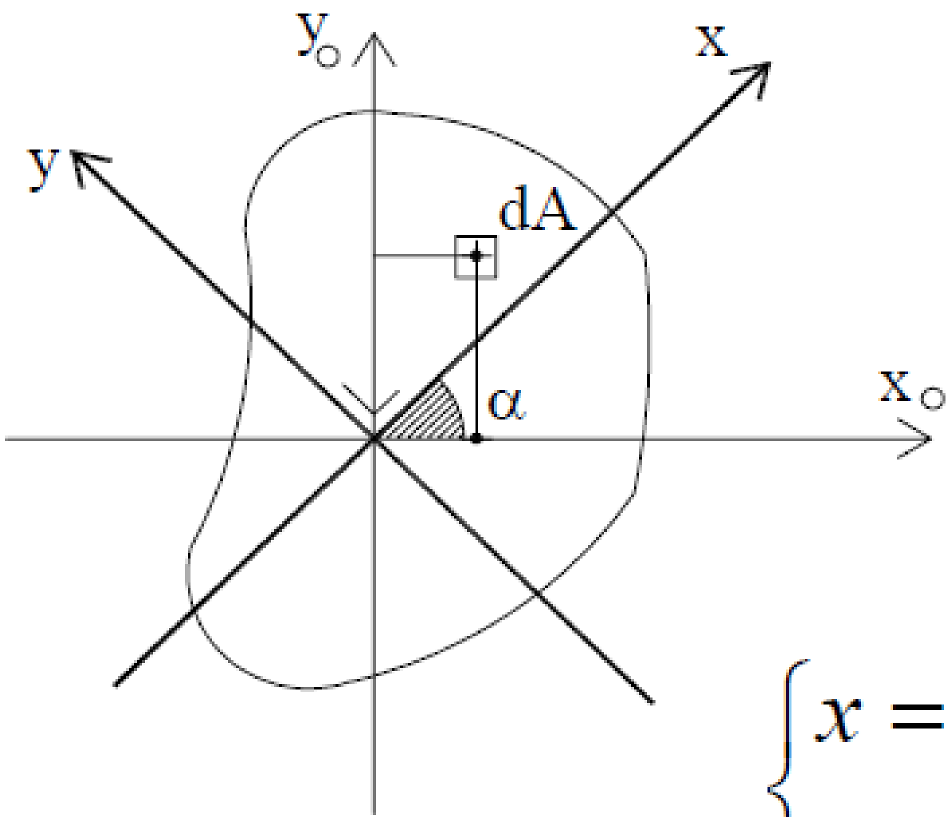
$$I_x = \frac{1}{12} b h^3$$

$$I_y = \frac{1}{12} h b^3$$

$$I_{xy} = \int_A xy \cdot dA$$

= 0 si x ou y axes de symétrie

Et si on a des axes génériques?



$$\begin{cases} x = x_0 \cos\alpha + y_0 \sin\alpha \\ y = -x_0 \sin\alpha + y_0 \cos\alpha \end{cases}$$

$$Ix = \int_A y^2 dA = \int_A \left(-x_0 \sin\alpha + y_0 \cos\alpha\right)^2 dA =$$

$$= \int_A \left(x_0^2 \sin^2 \alpha - 2x_0 y_0 \sin\alpha \cos\alpha + y_0^2 \cos^2 \alpha\right) dA$$

$$Ix = \sin^2 \alpha \cdot Iy_0 - 2\sin\alpha\cos\alpha \cdot Ix_0 y_0 + \cos^2 \alpha \cdot Ix_0$$

$$Iy = \cos^2 \alpha \cdot Iy_0 + 2\sin\alpha\cos\alpha \cdot Ix_0 y_0 + \sin^2 \alpha \cdot Ix_0$$

$$Ixy = \frac{1}{2}\sin 2\alpha\left(Ix_0 - Iy_0\right) + \cos 2\alpha \cdot Ix_0 y_0$$

$$Ixy = 0$$

$$\frac{1}{2}\sin 2\alpha\left(Ix_0 - Iy_0\right) + \cos 2\alpha \cdot Ix_0 y_0 = 0$$

$$\frac{\sin 2\alpha}{\cos 2\alpha} = -\frac{2Ix_0 y_0}{Ix_0 - Iy_0}$$

$$tg2\alpha = -\frac{2Ix_0 y_0}{Ix_0 - Iy_0}$$

$$\alpha = \frac{1}{2} artg\left(-\frac{2Ix_0 y_0}{Ix_0 - Iy_0} \right)$$

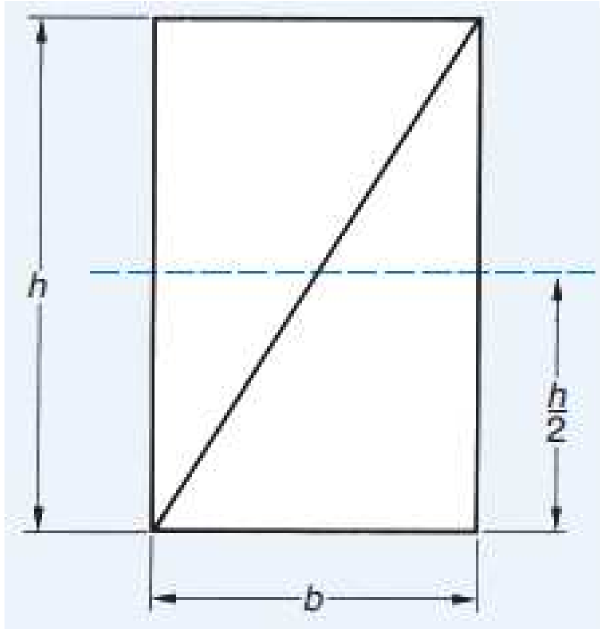
$$I_{MAX}$$

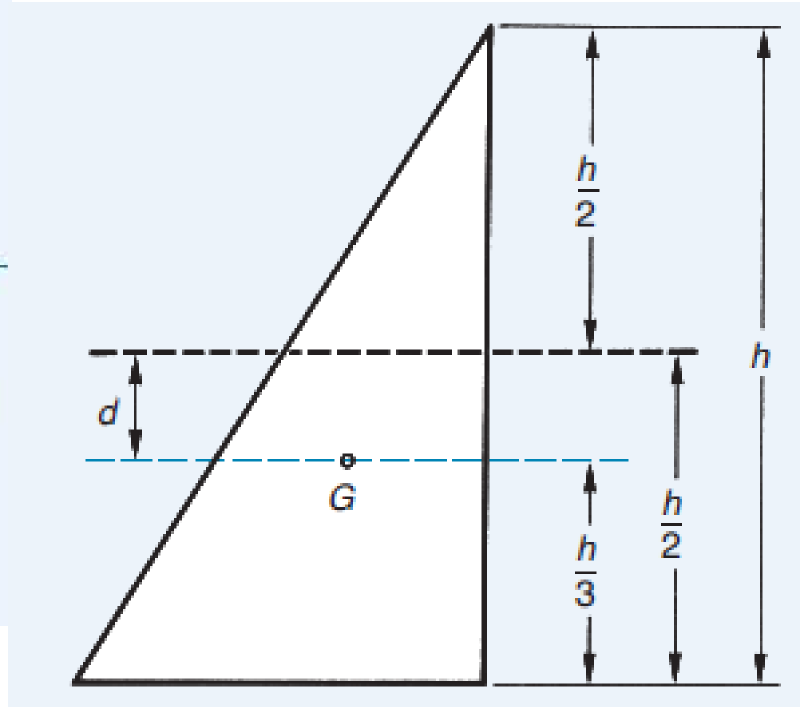
$$= \frac{Ix_0 + Iy_0}{2} \pm \sqrt{\left(\frac{Ix_0 - Iy_0}{2} \right)^2 + (Ix_0 y_0)^2}$$

$$I_{MIN}$$

Moment d'inertie de quelque figure géométrique

ATTENTION AUX AXES !!





$$I_{x_0} = \frac{b \cdot h^3}{12}$$ rectangle

$$I_x = \frac{b \cdot h^3}{24}$$ triangle

mais si j'utilise

$$I_{x_0} = I_x - A \cdot d^2$$

on a $$I_{x_0} = \frac{1}{36} \cdot b \cdot h^3$$